

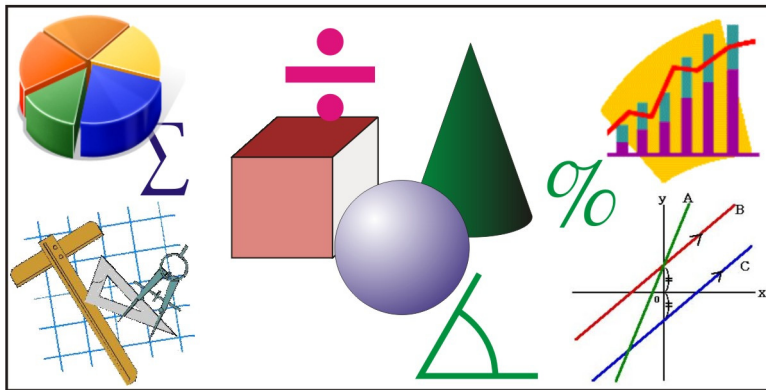
**New!**

**RAVEN'S GUIDE TO  
SASKATCHEWAN**

**PRE CALCULUS MATH 20**

**LINKED DIRECTLY TO NEW CURRICULUM REQUIREMENTS  
FROM THE WESTERN PROTOCOLS FOR 2008 AND BEYOND**

**STUDENT GUIDE AND  
RESOURCE BOOK**



**Key to Student Success  
with Grade 11 Mathematics**

**One of a series of publications by Raven Research Associates  
for Secondary and Elementary Mathematics**

**Alan R. Taylor, Ed. D.  
Bill Kokoskin, M.A.**

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## 7.4 The Discriminant

- In the section on graphing quadratic equations we showed the following examples where the graph intersected the x-axis in two locations, in one location, and in no locations.

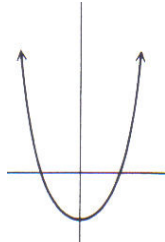


Figure 1

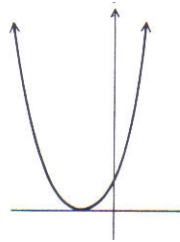


Figure 2

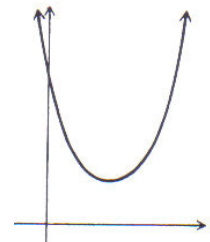


Figure 3

- In Figure 1 the equation has 2 real roots, in Figure 2 it has one double real root, and in Figure 3 it has no real roots (we could say it has 2 imaginary roots).
- We can tell what types of roots a quadratic equation has by taking a closer look at the quadratic formula below.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

### The Discriminant

- The expression  $b^2 - 4ac$ , which is under the radical sign in the quadratic formula, is called the **Discriminant (D)**. By looking at its value we can tell certain things about the roots of the equation. Some examples follow.

Equation	Solution Using the Formula	Value of Discriminant and Nature of the roots
<p>Example 1</p> $4x^2 - 12x + 9 = 0$	$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4 \cdot 4 \cdot 9}}{2 \cdot 4}$ $= \frac{12 \pm \sqrt{144 - 144}}{8} = \frac{12 \pm \sqrt{0}}{8}$ $= \frac{12}{8} = \frac{3}{2}$	$b^2 - 4ac = 0$ $D = 0$ <p>The root is <math>\frac{3}{2}</math></p> <p>The roots are real and equal.</p>
<p>Example 2</p> $2x^2 - x - 6 = 0$	$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \cdot 2 \cdot (-6)}}{2 \cdot 2}$ $x = \frac{1 \pm \sqrt{1 - (-48)}}{4} = \frac{1 \pm \sqrt{49}}{4}$ $x = \frac{1+7}{4}, \frac{1-7}{4}$ $x = 2, -\frac{3}{2}$	$D = 49$ $b^2 - 4ac > 0$ <p>The roots are 3 and <math>-\frac{3}{2}</math></p> <p>The two roots are real and unequal</p>
<p>Example 3</p> $x^2 + x + 3 = 0$	$x = \frac{-1 \pm \sqrt{1^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$ $x = \frac{-1 \pm \sqrt{1 - 12}}{2} = \frac{-1 \pm \sqrt{-11}}{2}$	$D = -11$ $b^2 - 4ac < 0$ <p>We can't find the square root of a negative number in the real number system.</p> <p>There are no real roots.</p>

## Nature of the Roots of a Quadratic Equation

- Next we will make the following generalizations about the value of the discriminant ( $D = b^2 - 4ac$ ) and the nature of the roots of a quadratic equation.

For the quadratic equation $ax^2 + bx + c$ , $a \neq 0$		
$D = b^2 - 4ac > 0$	▪ Roots are real and unequal	If $a$ , $b$ , and $c$ are rational and $D \geq 0$ When $b^2 - 4ac$ is a perfect square When $b^2 - 4ac$ is not a perfect square
$D = b^2 - 4ac = 0$	▪ Roots are real and equal	
$D = b^2 - 4ac < 0$	▪ There are no real roots	
		▪ The roots are rational
		▪ The roots are irrational

e.g. Find the value of each discriminant and describe the nature of the roots.

- $x^2 - 6x - 3 = 0$ 
  - $D = b^2 - 4ac$
  - $D = (-6)^2 - 4(1)(-3) = 36 + 12 = 48$
  - $D > 0$ , so there are 2 real and unequal roots
  - Since  $a$ ,  $b$ ,  $c$  are rational and  $D$  is not a perfect square the roots are irrational
- $2x^2 + x + 4 = 0$ 
  - $D = b^2 - 4ac$
  - $D = (1)^2 - 4(2)(4) = 1 - 32 = -31$
  - $D < 0$ , so there are no real roots
- $9x^2 - 6x + 1 = 0$ 
  - $D = b^2 - 4ac$
  - $D = (-6)^2 - 4(9)(1) = 36 - 36 = 0$
  - $D = 0$ , so there are 2 real and equal roots (a double root)
  - Since  $a$ ,  $b$ ,  $c$  are rational and  $D$  is a perfect square the roots are rational
- $2x^2 - \sqrt{3}x - 5 = 0$ 
  - $D = b^2 - 4ac$
  - $D = (-\sqrt{3})^2 - 4(2)(-5) = 3 + 40 = 43$
  - $D > 0$ , so there are 2 real and unequal roots
  - Since  $D$  is not a perfect square the roots are irrational
- $2x - \frac{1}{3}x^2 = 6$ 
  - $D = b^2 - 4ac$
  - $D = (2)^2 - 4(-\frac{1}{3})(-6) = 4 - 8 = -4$
  - $D < 0$ , so there are no real roots

## Exercises 7.4 The Discriminant

1. Find the value of the discriminant for each equation and give the nature of its roots.

a.  $3x^2 - x - 1 = 0$

b.  $-2x^2 + x + 1 = 0$

c.  $1 - 5x^2 = 8$

d.  $-3x^2 - x - 5 = 0$

e.  $2x(1 - 2x) = x - 5$

f.  $\sqrt{3}x^2 - \sqrt{5}x + \sqrt{3} = 0$

g.  $3t^2 - \sqrt{5}t = 7$

h.  $\sqrt{2}x^2 - 3x - \sqrt{8} = 0$

i.  $\frac{1}{x+1} = 2 + 3x$

j.  $\frac{r-3}{r} = \frac{2r+1}{3}$

2. Determine all values of  $k$  for which each equation will have the indicated number of roots.

a.  $x^2 + kx + 2$ ; 1 real double root

b.  $x^2 - kx + 2$ ; no real roots

c.  $2x^2 - 3x + k$ ; 2 real unequal roots

d.  $-3x^2 - kx - 5 = 0$ ; 1 real double root

3. In each of the following determine k to that (i) there are two real roots, (ii) there is one double real root, and (iii) there are no real roots.

a.  $x^2 - kx + 5 = 0$  (i) there are two real roots \_\_\_\_\_

(ii) there is one double real root \_\_\_\_\_

(iii) there are no real roots \_\_\_\_\_

b.  $kx^2 + 4x + 2 = 0$  (i) there are two real roots \_\_\_\_\_

(ii) there is one double real root \_\_\_\_\_

(iii) there are no real roots \_\_\_\_\_

c.  $x^2 + 2x = 1 - 2k$  (i) there are two real roots \_\_\_\_\_

(ii) there is one double real root \_\_\_\_\_

(iii) there are no real roots \_\_\_\_\_

d.  $2x^2 + kx + k = 0$  (i) there are two real roots \_\_\_\_\_

(ii) there is one double real root \_\_\_\_\_

(iii) there are no real roots \_\_\_\_\_

4. For what value(s) of k would the following equation have one double real root?

a.  $kx^2 - x + 2 = 0$

b.  $x^2 - kx + 2 = 0$