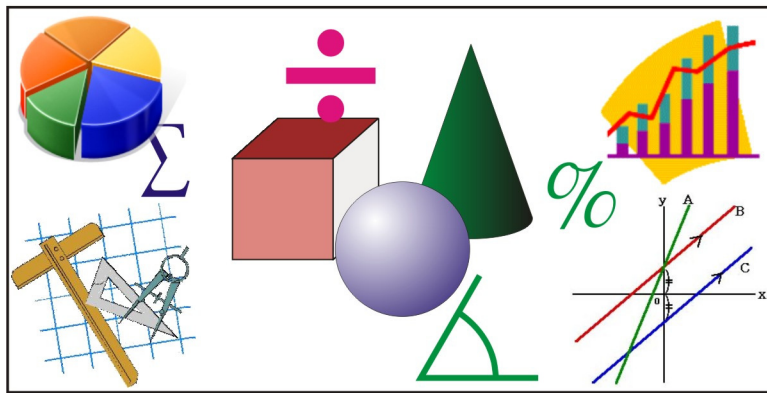


# RAVEN'S CORE MATHEMATICS GRADE 9

## MODIFIED PROGRAM

(Designed for the Western Provinces and the Territories)

### STUDENT GUIDE AND RESOURCE BOOK



## The Key to Student Success

One of a series of publications by Raven Research Associates  
for Secondary and Elementary Mathematics

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## Introduction

This book is intended to provide students taking **Literacy Foundations Level 6 Mathematics** with a practical resource designed to enhance success. It is linked to the provincial mathematics curriculum and designed by experienced teachers of mathematics to provide students with greater success in these courses. It includes the following features:

- Clear Descriptions of the Key Concepts
- Numerous Examples with Step-by-Step Solutions
- Many Practice Exercises to Reinforce Understanding and Application
- Review Exercises with a Range of Difficulty Levels
- All Answers which are listed at the Back of the Book
- Produced by Experienced Teachers of Mathematics
- Attractively Bound and Formatted for Clarity and Ease of Access

The content areas listed in the Table of Contents, shown in the next page, are linked to the following Prescribed Learning Outcomes for Literacy Foundations Level 6 Mathematics. .

### NUMBER

<p>A1 demonstrate an understanding of perfect square and square root concretely, pictorially, and symbolically</p> <p>A2 determine the square root of positive whole and rational numbers that are perfect squares</p> <p>A3 determine, using technology, the approximate square root of positive rational numbers that are non-perfect squares and justify their reasonableness</p> <p>A4 demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:</p>	<ul style="list-style-type: none"> <li>- representing repeated multiplication using powers</li> <li>- using patterns to show that a power with an exponent of zero is equal to one</li> <li>- solving problems involving powers</li> </ul> <p>A5 demonstrate an understanding of rational numbers by</p> <ul style="list-style-type: none"> <li>- comparing and ordering rational numbers</li> <li>- solving problems that involve arithmetic operations on rational numbers, with or without technology</li> </ul> <p>A6 explain and apply the order of operations, including exponents, with or without technology</p>
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### PATTERNS AND RELATIONS

<p><b>Patterns</b></p> <p>B1 determine if the relationship between two variables is linear and justify the reasoning</p> <p>B2 generate a pattern from a problem using linear equations and verify by substitution</p> <p>B3 graph linear relations, analyse the graph, and interpolate or extrapolate from the graph to solve problems</p> <p><b>Variables and Equations</b></p> <p><i>It is expected that students will:</i></p> <p>B4 model and solve problems using linear equations of the form</p> <ul style="list-style-type: none"> <li>- <math>a\chi = b</math>, <math>\chi/a = b</math>, <math>a \neq 0</math>, <math>a\chi + b = c</math></li> <li>- <math>\chi/a + b = d</math>, <math>a \neq 0</math>, <math>a(\chi + b) = c</math>, <math>a\chi + b = c\chi + d</math></li> <li>- <math>a(b\chi + c) = d(e\chi + f)</math>, <math>- a/\chi = b</math>, <math>\chi \neq 0</math></li> </ul> <p>where <math>a</math>, <math>b</math>, <math>c</math>, <math>d</math>, <math>e</math>, and <math>f</math> are rational numbers</p>	<p>B5 solve single variable linear inequalities with rational coefficients</p> <p>B6 demonstrate an understanding of polynomials (of degree less than or equal to 2) by</p> <ul style="list-style-type: none"> <li>- identifying the variables, degree, number of terms and coefficients, including the constant term of a given simplified polynomial expression</li> <li>- describing a situation for a given first-degree polynomial expression</li> <li>- matching equivalent polynomial expressions given in simplified form (e.g., <math>4x - 3x^2 + 2</math> is equivalent to <math>-3x^2 + 4x + 2</math>)</li> </ul> <p>B7 add and subtract polynomial expressions (of degree less than or equal to 2)</p> <p>B8 multiply and divide polynomial expressions by monomials (of degree less than or equal to 2)</p>
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### SHAPE AND SPACE

<p><b>Measurement</b></p> <p>C1 develop and apply the Pythagorean theorem to solve problems</p> <p><b>3-D Objects and 2-D Shapes</b></p> <p>C2 explain and describe polygons and polyhedra in terms of their edges, faces, and vertices</p>	<p>C3 determine the surface area of:</p> <ul style="list-style-type: none"> <li>- right rectangular prisms</li> <li>- right triangular prisms</li> <li>- right cylinders</li> <li>- composite 3-D objects to solve problems</li> </ul>
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## *SAMPLE EXERCISE*

### 1.6.2 Comparing and Ordering Fractions and Decimals

#### Review of Fractions and Decimals

- a **Proper Fraction** is a common fraction in which the numerator is smaller than the denominator  
e.g.  $\frac{1}{100}$ ,  $\frac{3}{5}$ ,  $\frac{11}{13}$  - these fractions are all between 0 and 1
- an **Improper Fraction** is a common fraction in which the numerator is larger than the denominator  
e.g.  $\frac{5}{3}$ ,  $\frac{3}{2}$ ,  $\frac{7}{4}$  - these fractions are all greater than 1
- the number 1 can be written as a fraction in a variety of ways and could be called a **Fraction of One** e.g.  $1 = \frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \dots$

#### Mixed Numbers

- a **Mixed Number** is greater than 1 and is a combination of a whole number and a proper fraction e.g.  $2\frac{1}{2}$ ,  $5\frac{3}{4}$ ,  $1\frac{1}{10}$
- all improper fractions can be written as mixed numbers and vice versa

#### Converting Mixed Numbers and Improper Fractions

- One of the methods we looked at earlier in converting between mixed numbers and improper fractions was to rewrite them using expanded form.  
e.g. 1. From an improper fraction to a mixed number

$$\frac{5}{3} = \frac{3}{3} + \frac{2}{3} = 1 + \frac{2}{3} = 1\frac{2}{3}$$

2. From a mixed number to an improper fraction

$$2\frac{1}{5} = 2 + \frac{1}{5} = \frac{10}{5} + \frac{1}{5} = \frac{11}{5}$$

or multiply the denominator times the whole number and add the numerator, over the denominator

$$\text{e.g. } 2\frac{1}{5} = \frac{5 \times 2 + 1}{5} = \frac{11}{5}$$

### Converting Fractions and Decimals

- In the previous section we converted **fractions to decimals** using equivalent fractions with denominators that are powers of 10 or else by dividing the denominator into the numerator.

$$\text{e.g. } \frac{3}{100} = 0.03, \frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4 \text{ and } \frac{3}{20} = 20 \overline{)3} = 0.15$$

- We converted terminating **decimals to fractions** by just reading off the place value.

$$\text{e.g. } 0.031 = \frac{31}{1000}, 0.17 = \frac{17}{100}, 2.09 = 2\frac{9}{100}$$

### Examples with Solutions

- Write each decimal as a fraction or mixed number in simplest form.

a. 0.13 *13 hundredths,*  $0.13 = \frac{13}{100}$

b. 0.007 *7 thousandths,*  $0.007 = \frac{7}{1000}$

c. 0.25 *25 hundredths,*  $0.25 = \frac{25}{100} = \frac{1}{4}$

d. 3.8 *3 and 8 tenths,*  $3.8 = 3\frac{8}{10} = 3\frac{4}{5}$

e. 21.04 *21 and 4 hundredths =*  $21\frac{4}{100} = 21\frac{1}{25}$

- Write each fraction as a decimal or a mixed number decimal.

a.  $\frac{19}{1000}$  *19 thousandths,* 0.019

b.  $2\frac{3}{100}$  *2 and 3 hundredths,* 2.03

c.  $\frac{7}{25}$   $\frac{7}{25} = \frac{7 \times 4}{25 \times 4} = \frac{28}{100} = 0.28$  or  $\frac{7}{25} = 25 \overline{)7} = 0.28$

d.  $\frac{11}{30}$   $\frac{11}{30} = 30 \overline{)11} = 0.3\bar{6}$

e.  $3\frac{4}{11}$

$3 \text{ and } \frac{4}{11} = 3 + 11\overline{)4} = 3.\overline{36}$

3. Write each improper fraction as a mixed number.

a.  $\frac{7}{5}$

$\frac{7}{5} = \frac{5}{5} + \frac{2}{5} = 1\frac{2}{5}$

b.  $\frac{11}{2}$

$\frac{11}{2} = \frac{10}{2} + \frac{1}{2} = 5\frac{1}{2}$

c.  $\frac{4}{3}$

$\frac{4}{3} = \frac{3}{3} + \frac{1}{3} = 1\frac{1}{3}$

4. Write each mixed number as an improper fraction.

a.  $3\frac{1}{3}$

$3\frac{1}{3} = \frac{3 \times 3 + 1}{3} = \frac{10}{3}$

b.  $12\frac{2}{5}$

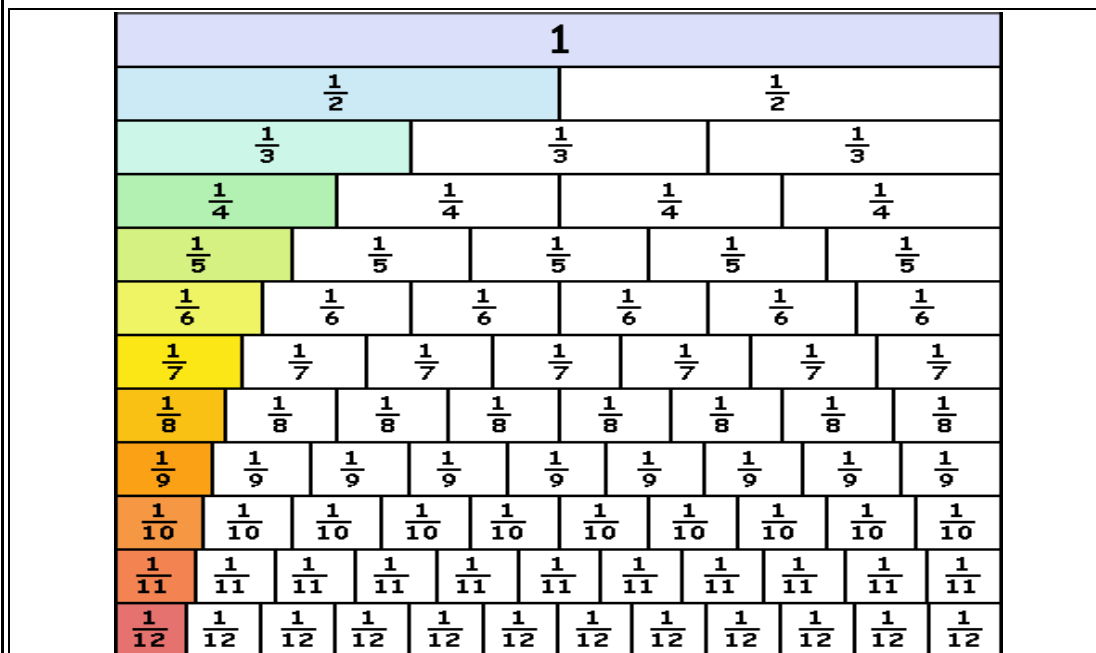
$12\frac{2}{5} = \frac{5 \times 12 + 2}{5} = \frac{62}{5}$

c.  $2\frac{3}{1000}$

$2\frac{3}{1000} = \frac{1000 \times 2 + 3}{1000} = \frac{2003}{1000}$

## Comparing and Ordering Fractions and Decimals

- Sometimes we need to compare two fractions to discover which is larger or smaller. One way of doing this is to use a fraction wall.



- The **fraction wall** above shows a value of 1 divided several times into sets of equal parts. By looking at the amount of space taken up by each part, we can tell which fractions are larger and which ones are smaller.

- We will use the following symbols to show the relationship between numbers:

“>” *greater than* e.g.  $3 > 2$ ; “<” *less than* e.g.  $5 < 8$ ; and “=” e.g.  $2 = \frac{6}{3}$

Looking at the fraction wall shown above we can see the following relations by the size of each of the corresponding parts.

$$\frac{1}{2} > \frac{1}{3}, \quad \frac{1}{8} < \frac{1}{7}, \quad \frac{3}{6} = \frac{4}{8}, \quad \frac{3}{7} > \frac{2}{5}$$

### Ways to Compare Fractions

- There are two easy ways to compare fractions: using decimals or using the same denominator. We will convert to decimals in the example below

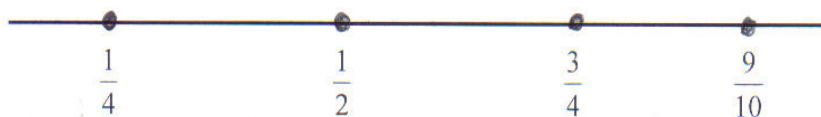
e.g. 1. Which is bigger  $\frac{3}{8}$  or  $\frac{5}{12}$ ? (you may want to use a calculator for this)

(i) Write each as a decimal  $8 \overline{)3} = 0.375$  and  $12 \overline{)5} = 0.416\bar{6}$

(ii) Using place value we can see that  $\frac{5}{12} = 0.416\bar{6}$  is larger.

### Location on the Number Line

- We could also look at location on the number line to compare fractions in order of size. If one fraction is located to the left of another, it is smaller or if one fraction is located to the right of another it is larger.



- Looking at the number line above we can see that  $\frac{1}{4} < \frac{1}{2}$  or that  $\frac{9}{10} > \frac{3}{4}$

To **Order Fractions** from Smallest to Largest or Vice Versa

- rewrite each fraction as a decimal fraction first
- use place value to order them
- re-write the fractions in their original forms

### Examples with Solutions

- Compare each pair of fractions. Which is largest or smallest?

a.  $\frac{1}{2}$  and  $\frac{3}{5}$

b.  $\frac{2}{9}$  and  $\frac{1}{4}$

c.  $\frac{3}{4}$  and  $\frac{7}{10}$

Answer

a.  $\frac{1}{2} = 0.5$  and  $\frac{3}{5} = 0.6$ :  $\frac{1}{2} < \frac{3}{5}$

b.  $\frac{2}{9} = 0.2\bar{2}$  and  $\frac{1}{4} = 0.25$ :  $\frac{2}{9} < \frac{1}{4}$

c.  $\frac{3}{4} = 0.75$  and  $\frac{7}{10} = 0.7$ :  $\frac{3}{4} > \frac{7}{10}$

2. Arrange each set of fractions from **smallest to largest**.

a.  $\frac{2}{3}, \frac{1}{4}, \frac{9}{10}, \frac{1}{2}$

b.  $\frac{1}{4}, \frac{2}{11}, \frac{2}{10}, \frac{1}{2}$

*Write as decimals first to order by place value, then in original form*

a.  $\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{9}{10}$

b.  $\frac{2}{11}, \frac{2}{10}, \frac{1}{4}, \frac{1}{2}$

3. Arrange each set of numbers from **largest to smallest**.

a.  $1\frac{1}{4}, 1.3, \frac{9}{5}, \frac{3}{2}, 0.8$

b.  $\frac{4}{3}, 1.2, \frac{5}{4}, 1\frac{2}{9}$

*Write as decimals first to order by place value, then in original form*

a.  $\frac{9}{5}, \frac{3}{2}, 1.3, 1\frac{1}{4}, 0.8$

b.  $\frac{4}{3}, \frac{5}{4}, 1\frac{2}{9}, 1.2$

4. The following numbers are supposed to be in order from **smallest to largest**. Which number is out of place?

$0.2, \frac{1}{4}, \frac{3}{5}, 0.5$

$\frac{3}{5} = 0.6$ , *it is larger than 0.5*

### Exercises 1.6.2

1. Write each decimal as a fraction or mixed number in simplest form.

a. 0.35

b. 0.101

c. 1.03

d. 5.005

e.  $4.7\overline{77}$

f.  $6.09\overline{09}$

2. Write each fraction as a decimal or a mixed number decimal.

a.  $\frac{7}{1000}$

b.  $2\frac{1}{3}$

c.  $\frac{7}{20}$

d.  $\frac{9}{25}$

e.  $4\frac{8}{11}$

f.  $\frac{9}{16}$

3. Write each improper fraction as a mixed number.

a.  $\frac{11}{7}$

b.  $\frac{9}{5}$

c.  $\frac{7}{3}$

d.  $\frac{25}{7}$

e.  $\frac{21}{10}$

f.  $\frac{50}{3}$

4. Write each mixed number as an improper fraction.

a.  $2\frac{1}{7}$

b.  $5\frac{3}{5}$

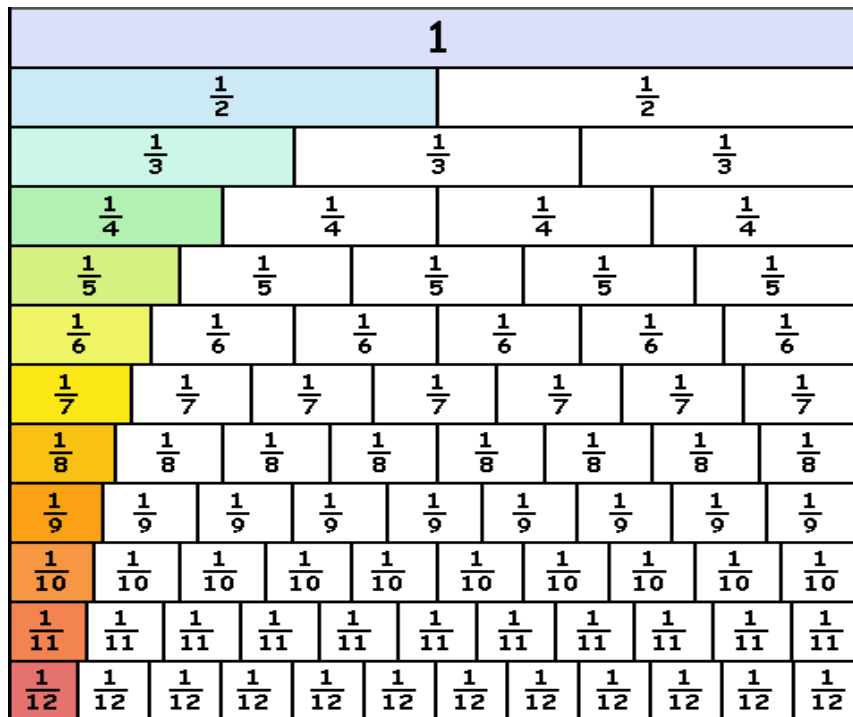
c.  $10\frac{3}{11}$

d.  $21\frac{3}{10}$

e.  $11\frac{10}{11}$

f.  $1000\frac{1}{10}$

Refer to the fraction wall shown below to determine the relationship between each pair of fractions in Question #5. The first one (#5a) is done for you.



Refer to the fraction wall above to determine the relationship between each pair of fractions. Use the following symbols to show it.

“>” *greater than* ; “<” *less than* ; and “=” *equal*

5. Compare each pair of fractions. Which fraction is largest or smallest, or are they equal?

a.  $\frac{1}{2} > \frac{1}{3}$

b.  $\frac{1}{8} \frac{1}{7}$

c.  $\frac{2}{3} \frac{4}{6}$

d.  $\frac{4}{9} \frac{3}{7}$

e.  $\frac{1}{2} \frac{4}{8}$

f.  $\frac{3}{7} \frac{2}{5}$

g.  $\frac{3}{5} \frac{4}{7}$

h.  $\frac{7}{11} \frac{6}{10}$

6. Arrange each set of fractions from **smallest to largest**.

a.  $\frac{1}{2}, \frac{1}{3}, \frac{3}{4}, \frac{2}{3}$

b.  $\frac{4}{5}, \frac{1}{10}, \frac{1}{11}, \frac{2}{5}$

c.  $\frac{3}{25}, \frac{1}{4}, \frac{3}{10}, \frac{1}{5}$

d.  $\frac{5}{6}, \frac{2}{3}, \frac{3}{4}, \frac{3}{10}$

e.  $\frac{5}{4}, \frac{9}{10}, \frac{6}{5}, \frac{19}{20}$

f.  $\frac{2}{3}, \frac{5}{6}, \frac{3}{12}, \frac{6}{5}$

7. Arrange each set of fractions, decimals, and mixed numbers from **largest to smallest**.

a.  $\frac{4}{5}, 1\frac{1}{4}, 0.5, 1.3$

b.  $0.4, \frac{3}{10}, \frac{4}{5}, \frac{10}{3}, 1.2$

c.  $2.1, \frac{11}{10}, 0.9, \frac{49}{50}$

d.  $\frac{3}{4}, 0.7, \frac{3}{8}, 1.1$

e.  $\frac{3}{25}, 0.1, \frac{3}{2}, 1.1$

f.  $\frac{4}{5}, \frac{5}{4}, 1\frac{3}{4}, 1.2$

8. The following numbers are supposed to be in order from **smallest to largest**. Which number is out of place?

a.  $1.2, \frac{5}{4}, 1.09, \frac{8}{3}$

b.  $\frac{6}{5}, 1\frac{1}{2}, 1.3, \frac{5}{3}$

c.  $\frac{4}{5}, 1\frac{1}{5}, 1.5, 1\frac{1}{4}$

d.  $2.25, \frac{10}{4}, 2\frac{1}{5}, 2.75$

e.  $\frac{3}{10}, 0.25, \frac{3}{5}, 0.75$

f.  $\frac{1}{5}, 0.25, \frac{3}{10}, 0.19$