

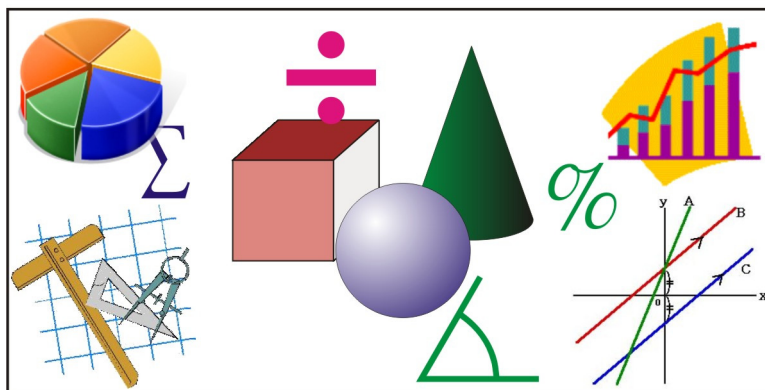
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RAVEN'S WNCP GRADE 11 MATHEMATICS

**BC – Pre Calculus Math 11
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STUDENT GUIDE AND RESOURCE BOOK



The Key to Student Success

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**Raven Research Associates
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SAMPLE EXERCISE

2.A Writing Radicals in Simplest Form

- A radical expression representing a square root is in its **simplest form (or simple radical form)** when the following properties are in place.
 - 1. The radicand contains no integer (other than 1) that is a perfect square
 - 2. No fraction is under the radical sign
 - 3. No radical is in the denominator
- 1. In Chapter 1 we simplified radicals by making sure that **the radicand contained no perfect squares other than 1** remained under the radical sign.

e.g. Simplify

$$(i) \quad \sqrt{49} = \sqrt{7 \cdot 7 \cdot 1} = 7\sqrt{1} = 7$$

$$(ii) \quad \sqrt{12} = \sqrt{2 \cdot 2 \cdot 3} = 2\sqrt{3}$$

$$(iii) \quad \sqrt{99} = \sqrt{3 \cdot 3 \cdot 11} = 3\sqrt{11}$$

$$(iv) \quad \sqrt{8a^2} = \sqrt{2 \cdot 2 \cdot 2 \cdot a \cdot a} = 2a\sqrt{2}$$

- 2. Also, in the Chapter 1 we **simplified radicals containing fractions** using properties of division.

e.g. Simplify

$$(i) \quad \frac{\sqrt{15}}{\sqrt{3}} = \sqrt{\frac{15}{3}} = \sqrt{\frac{3 \cdot 5}{3}} = \sqrt{\frac{5}{1}} = \sqrt{5}$$

$$(ii) \quad \frac{\sqrt{3b^3}}{\sqrt{3b}} = \sqrt{\frac{3 \cdot b \cdot b \cdot b}{3 \cdot b}} = \sqrt{b^2} = b$$

- *However, we did not concern ourselves in Chapter 1 with expressions containing a radical in a denominator. e.g. $\frac{\sqrt{6}}{\sqrt{18}} = \sqrt{\frac{6}{6 \cdot 3}} = \sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$. So, according to property #3, this is not in simple radical form.*

▪ 3. **No radical should be left in the denominator**

In a fraction involving radicals, we prefer to have a whole number denominator. When we have a fraction where the denominator includes a radical, we change the denominator to a rational number (this is called **rationalizing the denominator**).

How do we do this with a single (monomial) term in the denominator?

- **First**, recall that when you multiply any number or expression by 1 that its value remains the same. **Second**, recall that 1 is a rational number that can be written in different ways.

e.g. $1 = \frac{1}{1} = \frac{5}{5} = \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt[3]{4}}{\sqrt[3]{4}}$, etc. , Note: any number written as a fraction with the same (non-zero) numerator and denominator is equal to 1.

- So, to **rationalize a denominator** in a fraction, multiply the numerator and denominator by an expression equal to 1 that clears the radical sign in the denominator and cancel like factors.

e.g. Rationalize the following denominators.

$$(i) \quad \frac{1}{\sqrt{3}} = \frac{1 \cdot \sqrt{3}}{\sqrt{3} \cdot \sqrt{3}} = \frac{\sqrt{3}}{3} \text{ or } \frac{1}{3} \sqrt{3}$$

$$(ii) \quad \frac{\sqrt{2b}}{\sqrt{3b}} = \frac{\sqrt{2b} \cdot \sqrt{3b}}{\sqrt{3b} \cdot \sqrt{3b}} = \frac{\sqrt{2b \cdot 3b}}{3b} = \frac{b\sqrt{6}}{3b} = \frac{\sqrt{6}}{3}$$

Simplifying Radicals with Other than Square Roots

- In addition to radicals with square roots, radicals with other roots can be simplified in a similar way.

e.g. Simply each of the following.

$$(i) \quad \sqrt[3]{64} = \sqrt[3]{4 \cdot 4 \cdot 4} = 4$$

$$(ii) \quad \sqrt[4]{16a^5} = \sqrt[4]{(2 \cdot 2 \cdot 2 \cdot 2) \cdot (a \cdot a \cdot a \cdot a) \cdot a} = 2a\sqrt[4]{a}$$

$$(iii) \quad \frac{1}{\sqrt[3]{2}} = \frac{1 \cdot \sqrt[3]{4}}{\sqrt[3]{2} \cdot \sqrt[3]{4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{2 \cdot 2 \cdot 2}} = \frac{\sqrt[3]{4}}{2} \text{ or } \frac{1}{2} \sqrt[3]{4}$$

Examples with Solutions

Simplify

Solution

1. $\sqrt{8y^3}$

$$\sqrt{8y^3} = \sqrt{2 \cdot 2^2 \cdot y \cdot y^2} = 2y\sqrt{2y}$$

2. $\sqrt[3]{27b}$

$$\sqrt[3]{27b} = \sqrt[3]{3^3 b} = 3\sqrt[3]{b}$$

3. $\frac{5}{\sqrt{3}}$

$$\frac{5}{\sqrt{3}} = \frac{5}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3} \text{ or } \frac{5}{3}\sqrt{3}$$

4. $\sqrt{\frac{3}{2}}$

$$\begin{aligned}\sqrt{\frac{3}{2}} &= \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{\sqrt{6}}{2} \text{ or } \frac{1}{2}\sqrt{6}\end{aligned}$$

5. $\sqrt[3]{\frac{7}{2}}$

$$\sqrt[3]{\frac{7}{2}} = \frac{\sqrt[3]{7}}{\sqrt[3]{2}} = \frac{\sqrt[3]{7} \cdot \sqrt[3]{2^2}}{\sqrt[3]{2} \cdot \sqrt[3]{2^2}} = \frac{\sqrt[3]{28}}{2}$$

6. $\sqrt[4]{\frac{5}{3}}$

$$\sqrt[4]{\frac{5}{3}} = \frac{\sqrt[4]{5}}{\sqrt[4]{3}} = \frac{\sqrt[4]{5} \cdot \sqrt[4]{3^3}}{\sqrt[4]{3} \cdot \sqrt[4]{3^3}} = \frac{\sqrt[4]{135}}{3}$$

7. $\sqrt[3]{\frac{2}{4b}}$

$$\sqrt[3]{\frac{2}{4b}} = \frac{\sqrt[3]{2}}{\sqrt[3]{4b}} = \frac{\sqrt[3]{2} \cdot \sqrt[3]{2b^2}}{\sqrt[3]{4b} \cdot \sqrt[3]{2b^2}} = \frac{\sqrt[3]{4b^2}}{2b}$$

Exercises 2.A

Simplify and write each of the following in simple radical form.

1. $\frac{1}{\sqrt{2}}$

2. $\frac{1}{\sqrt{5}}$

3. $\frac{2}{\sqrt{5}}$

4. $\frac{3}{\sqrt{6}}$

5. $\frac{\sqrt{2}}{\sqrt{3}}$

6. $\sqrt{\frac{3}{2}}$

7. $\frac{5\sqrt{2}}{2\sqrt{5}}$

8. $\frac{20\sqrt{6}}{4\sqrt{3}}$

9. $\frac{12\sqrt{3}}{3\sqrt{2}}$

10. $\frac{\sqrt{3}+2}{\sqrt{3}}$

11. $\frac{2\sqrt{5}-3}{\sqrt{2}}$

12. $\frac{5\sqrt{8}-2\sqrt{3}}{\sqrt{6}}$

13. $\sqrt{\frac{4}{a}}$

14. $\sqrt{\frac{5}{9b^2}}$

15. $\sqrt{\frac{1}{xy}}$

16. $\sqrt[3]{\frac{5}{9}}$

17. $\sqrt[3]{\frac{1}{2}}$

18. $\sqrt[4]{\frac{5}{2}}$

19. $\sqrt[3]{\frac{16}{2y}}$

20. $\sqrt[3]{\frac{16}{4y^2}}$

21. $\sqrt[3]{\frac{2}{4a^2b^2}}$

22. $\sqrt[3]{\frac{1}{8x^3}}$