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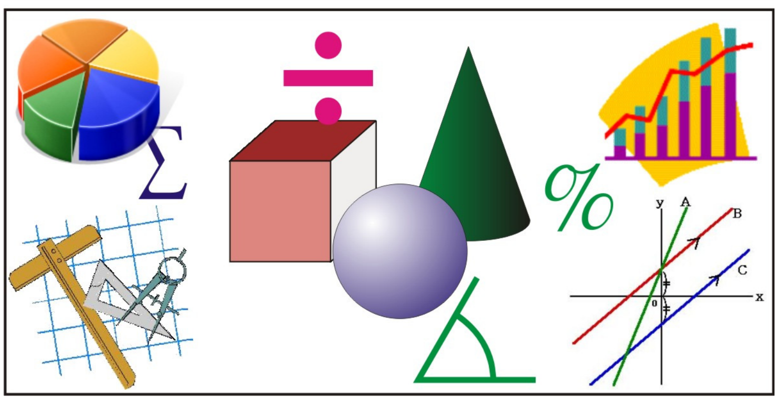

The Key to Student Success

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SAMPLE EXERCISE

2.A Writing Radicals in Simplest Form

- A radical expression representing a <u>square root</u> is in its **simplest form (or simple radical form)** when the following properties are in place.
  - 1. The radicand contains no integer (other than 1) that is a perfect square
  - 2. No fraction is under the radical sign
  - 3. No radical is in the denominator

- 1. In Chapter 1 we simplified radicals by making sure that **the radicand contained no perfect squares other than 1** remained under the radical sign.

e.g. Simplify

(i) $\sqrt{49} = \sqrt{7 \bullet 7 \bullet 1} = 7\sqrt{1} = 7$ (ii) $\sqrt{12} = \sqrt{2 \bullet 2 \bullet 3} = 2\sqrt{3}$

(iii) $\sqrt{99} = \sqrt{3 \bullet 3 \bullet 11} = 3\sqrt{11}$ (iv) $\sqrt{8a^2} = \sqrt{2 \bullet 2 \bullet 2 \bullet a \bullet a} = 2a\sqrt{2}$

- 2. Also, in the Chapter 1 we **simplified radicals containing fractions** using properties of division.

e.g. Simplify

(i) $\frac{\sqrt{15}}{\sqrt{3}} = \sqrt{\frac{15}{3}} = \sqrt{\frac{3 \bullet 5}{3}} = \sqrt{\frac{5}{1}} = \sqrt{5}$ (ii) $\frac{\sqrt{3b^3}}{\sqrt{3b}} = \sqrt{\frac{3 \bullet b \bullet b \bullet b}{3 \bullet b}} = \sqrt{b^2} = b$

- *However, we did not concern ourselves in Chapter 1 with expressions containing a radical in a denominator. e.g.* $\frac{\sqrt{6}}{\sqrt{18}} = \sqrt{\frac{6}{6 \bullet 3}} = \sqrt{\frac{1}{3}} = \frac{\sqrt{1}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$*. So, according to property #3, this is <u>not</u> in simple <u>radical</u> form.*

- 3. **No radical should be left in the denominator**
  In a fraction involving radicals, we prefer to have a whole number denominator. When we have a fraction where the denominator includes a radical, we change the denominator to a rational number (this is called **rationalizing the denominator.**

How do we do this with a single (monomial) term in the denominator?

- **First**, recall that when you multiply any number or expression by 1 that its value remains the same. **Second**, recall that 1 is a rational number that can be written in different ways.

  e.g. $1 = \frac{1}{1} = \frac{5}{5} = \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt[3]{4}}{\sqrt[3]{4}}, etc.$, *Note: any number written as a fraction with the same (non-zero) numerator and denominator is equal to 1.*

- So, to **rationalize a denominator** in a fraction, multiply the numerator and denominator by an expression equal to 1 that clears the radical sign in the denominator and cancel like factors.

  e.g. Rationalize the following denominators.

(i)
$$\frac{1}{\sqrt{3}} = \frac{1 \bullet \sqrt{3}}{\sqrt{3} \bullet \sqrt{3}} = \frac{\sqrt{3}}{3} \ or \ \frac{1}{3}\sqrt{3}$$

(ii)
$$\frac{\sqrt{2b}}{\sqrt{3b}} = \frac{\sqrt{2b} \bullet \sqrt{3b}}{\sqrt{3b} \bullet \sqrt{3b}} = \frac{\sqrt{2b \bullet 3b}}{3b} = \frac{b\sqrt{6}}{3b} = \frac{\sqrt{6}}{3}$$

<u>Simplifying Radicals with Other than Square Roots</u>

- In addition to radicals with square roots, radicals with other roots can be simplified in a similar way.

e.g. Simply each of the following.

(i) $\sqrt[3]{64} = \sqrt[3]{4 \bullet 4 \bullet 4} = 4$

(ii) $\sqrt[4]{16a^5} = \sqrt[4]{(2 \bullet 2 \bullet 2 \bullet 2) \bullet (a \bullet a \bullet a \bullet a) \bullet a} = 2a\sqrt[4]{a}$

(iii) $\frac{1}{\sqrt[3]{2}} = \frac{1 \bullet \sqrt[3]{4}}{\sqrt[3]{2} \bullet \sqrt[3]{4}} = \frac{\sqrt[3]{4}}{\sqrt[3]{2 \bullet 2 \bullet 2}} = \frac{\sqrt[3]{4}}{2} \ or \ \frac{1}{2}\sqrt[3]{4}$

Examples with Solutions

Simplify | *Solution*

1. $\sqrt{8y^3}$ $\quad$ $\sqrt{8y^3} = \sqrt{2 \bullet 2^2 \bullet y \bullet y^2} = 2y\sqrt{2y}$

2. $\sqrt[3]{27b}$ $\quad$ $\sqrt[3]{27b} = \sqrt[3]{3^3 b} = 3\sqrt[3]{b}$

3. $\dfrac{5}{\sqrt{3}}$ $\quad$ $\dfrac{5}{\sqrt{3}} = \dfrac{5}{\sqrt{3}} \; x \; \dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{5\sqrt{3}}{3} \; or \; \dfrac{5}{3}\sqrt{3}$

4. $\sqrt{\dfrac{3}{2}}$ $\quad$ $\sqrt{\dfrac{3}{2}} = \dfrac{\sqrt{3}}{\sqrt{2}} = \dfrac{\sqrt{3}}{\sqrt{2}} \; x \; \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{\sqrt{6}}{2} \; or \; \dfrac{1}{2}\sqrt{6}$

5. $\sqrt[3]{\dfrac{7}{2}}$ $\quad$ $\sqrt[3]{\dfrac{7}{2}} = \dfrac{\sqrt[3]{7}}{\sqrt[3]{2}} = \dfrac{\sqrt[3]{7} \bullet \sqrt[3]{2^2}}{\sqrt[3]{2} \bullet \sqrt[3]{2^2}} = \dfrac{\sqrt[3]{28}}{2}$

6. $\sqrt[4]{\dfrac{5}{3}}$ $\quad$ $\sqrt[4]{\dfrac{5}{3}} = \dfrac{\sqrt[4]{5}}{\sqrt[4]{3}} = \dfrac{\sqrt[4]{5} \bullet \sqrt[4]{3^3}}{\sqrt[4]{3} \bullet \sqrt[4]{3^3}} = \dfrac{\sqrt[4]{135}}{3}$

7. $\sqrt[3]{\dfrac{2}{4b}}$ $\quad$ $\sqrt[3]{\dfrac{2}{4b}} = \dfrac{\sqrt[3]{2}}{\sqrt[3]{4b}} = \dfrac{\sqrt[3]{2} \bullet \sqrt[3]{2b^2}}{\sqrt[3]{4b} \bullet \sqrt[3]{2b^2}} = \dfrac{\sqrt[3]{4b^2}}{2b}$

Exercises 2.A

Simplify and write each of the following in simple radical form.

1. $\dfrac{1}{\sqrt{2}}$

2. $\dfrac{1}{\sqrt{5}}$

3. $\dfrac{2}{\sqrt{5}}$

4. $\dfrac{3}{\sqrt{6}}$

5. $\dfrac{\sqrt{2}}{\sqrt{3}}$

6. $\sqrt{\dfrac{3}{2}}$

7. $\dfrac{5\sqrt{2}}{2\sqrt{5}}$

8. $\dfrac{20\sqrt{6}}{4\sqrt{3}}$

9. $\dfrac{12\sqrt{3}}{3\sqrt{2}}$

10. $\dfrac{\sqrt{3}+2}{\sqrt{3}}$

11. $\dfrac{2\sqrt{5}-3}{\sqrt{2}}$

12. $\dfrac{5\sqrt{8}-2\sqrt{3}}{\sqrt{6}}$

13. $\sqrt{\dfrac{4}{a}}$

14. $\sqrt{\dfrac{5}{9b^2}}$

15. $\sqrt{\dfrac{1}{xy}}$

16. $\sqrt[3]{\dfrac{5}{9}}$

17. $\sqrt[3]{\dfrac{1}{2}}$

18. $\sqrt[4]{\dfrac{5}{2}}$

19. $\sqrt[3]{\dfrac{16}{2y}}$

20. $\sqrt[3]{\dfrac{16}{4y^2}}$

21. $\sqrt[3]{\dfrac{2}{4a^2b^2}}$

22. $\sqrt[3]{\dfrac{1}{8x^3}}$